EAS4510

Basic Orbital Dynamics Using Desmos

Anil V. Rao

Department of Mechanical and Aerospace Engineering
University of Florida
Gainesville, FL 32611-6250

Objective

The objective of this assignment is to solve some simple problems in orbital dynamics and to use the online graphing calculator Desmos (`https:\\www.desmos.com`) to perform some graphing of the orbits. In all calculations below, it will be assumed that the gravitational parameter is that of the Earth and is assigned the value $\mu = 398600\ \text{km}^3/\text{s}^2$. In other words, it will be assumed that the Earth is the central body (that is, the spacecraft is in Earth orbit). Please note that the symbol "e" is reserved in Desmos to be the number e. In order to bypass this restriction, please use the symbol ϵ for the orbital eccentricity. A list of keyboard shortcuts in Desmos can be found at `http://s3.amazonaws.com/desmos/Desmos_Calculator_User_Guide.pdf` while the symbol ϵ can be found at `https://www.desmos.com/calculator/btn7ufopnw`

Question 1

Suppose that the orbit of a spacecraft has a parameter $p = 7000$ km and an eccentricity $e = 0.4$. Compute the semi-major axis for this orbit. Then, using this value of semi-major axis, construct a table in Microsoft Excel that contains the radius and speed for the values $\theta = (0, 45, 90, 135, 180, 225, 270, 315)$ deg. Display the table in the following form:

θ (deg)	r	v
0	–	–
45	–	–
90	–	–
135	–	–
180	–	–
225	–	–
270	–	–
315	–	–

Repeat the calculations (including the table) for $e = 0.6$. What do you observe about the speed at different points on the orbit?

Equations:

$$r = \frac{p}{1 + e\cos\theta}$$

$$v = \sqrt{\mu}\sqrt{\frac{2}{r} - \frac{1}{a}}$$

$$p = a(1 - e^2)$$

Question 2

Suppose that the semi-major axis of the orbit of a spacecraft is $a = 10000$ km and the eccentricity is $e = 0.6$. Compute the following quantities:

(a) the periapsis radius

(b) the apoapsis radius

(c) The specific mechanical energy

(d) The specific angular momentum

(e) the orbital period

Plot the orbit using Desmos.

Equations:

$$r_p = a(1 - e)$$
$$r_a = a(1 + e)$$
$$h = \sqrt{\mu p}$$
$$\mathcal{E} = -\frac{\mu}{2a}$$
$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}}$$

Question 3

It is desired to design an orbit with a period of 12 hours and an orbital eccentricity $e = 0.74$. Compute the following quantities:

(a) the periapsis radius

(b) the apoapsis radius

(c) the specific mechanical energy

(d) the parameter

(e) the specific angular momentum

Plot the orbit using Desmos.

Question 4

Suppose that the orbit of a spacecraft has an eccentricity $e = 0.5$ and a semi-major axis $a = 8000$ km. Suppose further that the true anomaly at a point on the orbit is $\theta = 30$ deg. Determine the time elapsed on the orbit since the last time the spacecraft was at periapsis and apoapsis. Plot the orbit using Desmos.

Equations:

$$t - t_p = \sqrt{\frac{a^3}{\mu}}\,(E - e\sin E)$$
$$E = 2\tan^{-1}\left(\sqrt{\tfrac{1-e}{1+e}}\tan\tfrac{\theta}{2}\right)$$
$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}}$$