EGM 3401

Theory Assignment #1

Spring 2017

Due Date: 10 February 2017

Question 1

Let $\mathbf{u}$ be a vector with constant magnitude. Prove that

$$\frac{{}^{\mathcal{A}}d\mathbf{u}}{dt} \cdot \mathbf{u} = 0$$

where $\mathcal{A}$ is an arbitrary reference frame.

Question 2

Let ${}^{\mathcal{A}}d\mathbf{b}/dt$ be the rate of change of vector $\mathbf{b}$ as viewed by an observer in reference frame $\mathcal{A}$. Explain why ${}^{\mathcal{A}}d\mathbf{b}/dt$ is observed the same in all reference frames.

Question 3

Let ${}^{\mathcal{A}}\boldsymbol{\omega}^{\mathcal{B}}$ be the angular velocity of reference frame $\mathcal{B}$ as viewed by an observer in reference frame $\mathcal{A}$. Prove that ${}^{\mathcal{A}}\boldsymbol{\omega}^{\mathcal{B}} = -{}^{\mathcal{B}}\boldsymbol{\omega}^{\mathcal{A}}$.

Question 4

Let ${}^{\mathcal{A}}\boldsymbol{\omega}^{\mathcal{B}}$ be the angular velocity of reference frame $\mathcal{B}$ as viewed by an observer in reference frame $\mathcal{A}$. Prove that

$$\frac{{}^{\mathcal{A}}d}{dt}\left({}^{\mathcal{A}}\boldsymbol{\omega}^{\mathcal{B}}\right) = \frac{{}^{\mathcal{B}}d}{dt}\left({}^{\mathcal{A}}\boldsymbol{\omega}^{\mathcal{B}}\right)$$

Question 5

Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be a right-handed orthonormal basis fixed in reference frame $\mathcal{B}$. Prove that ${}^{\mathcal{A}}d\mathbf{e}_i/dt,\ (i = 1,2,3)$ cannot have a component in the direction of $\mathbf{e}_i,\ (i = 1,2,3)$ (that is, for any value of i, the rate of change of $\mathbf{e}_i$ in reference frame $\mathcal{A}$ cannot have a component in the direction of $\mathbf{e}_i$).

Question 6

Let $\mathbf{u}_1$ and $\mathbf{u}_2$ be orthogonal unit vectors. Prove that

$$\frac{{}^{\mathcal{A}}d\mathbf{u}_1}{dt} \cdot \mathbf{u}_2 = -\mathbf{u}_1 \cdot \frac{{}^{\mathcal{A}}d\mathbf{u}_2}{dt}.$$

Question 7

Let $\mathbf{a}$ and $\mathbf{b}$ be vectors that lie in $\mathbb{E}^3$. Furthermore, consider the scalar product $\mathbf{a} \cdot \mathbf{b}$ between the vectors $\mathbf{a}$ and $\mathbf{b}$. Finally, let $\mathbf{E} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be a basis for $\mathbb{E}^3$. Show that the scalar product expressed in the basis $\mathbf{E}$ can be written as

$$\{\mathbf{a} \cdot \mathbf{b}\}_{\mathbf{E}} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

where

$$\{\mathbf{a}\}_{\mathbf{E}} = \left\{\begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array}\right\} \quad \text{and} \quad \{\mathbf{b}\}_{\mathbf{E}} = \left\{\begin{array}{c} b_1 \\ b_2 \\ b_3 \end{array}\right\}$$

are the matrix representations of the vectors $\mathbf{a}$ and $\mathbf{b}$, respectively, in the basis $\mathbf{E}$.

Question 8

Let **a** and **b** be vectors that lie in $\mathbb{E}^3$. Furthermore, consider the vector product between **a** and **b**. Finally, let $\mathbf{E} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be a basis for $\mathbb{E}^3$. Show that the vector product expressed in the basis **E** can be written in matrix-vector form as

$$\{\mathbf{a} \times \mathbf{b}\}_{\mathbf{E}} = \left\{ \begin{array}{ccc} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{array} \right\} \left\{ \begin{array}{c} b_1 \\ b_2 \\ b_3 \end{array} \right\},$$

where

$$\{\mathbf{a}\}_{\mathbf{E}} = \left\{ \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} \right\} \quad \text{and} \quad \{\mathbf{b}\}_{\mathbf{E}} = \left\{ \begin{array}{c} b_1 \\ b_2 \\ b_3 \end{array} \right\}$$

are the matrix representations of the vectors **a** and **b**, respectively, in the basis **E**.

Question 9

Let **a** and **b** be vectors in $\mathbb{E}^3$. Show that the triple vector product $\mathbf{a} \times (\mathbf{b} \times \mathbf{a})$ can be written as

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{a}) = (\mathbf{a} \cdot \mathbf{a})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{a}.$$